

## A Study on Fundamentals of $\Gamma$ - Soft Set Theory

Srinivasa Rao T<sup>1</sup>, Srinivasa Kumar B<sup>2</sup> and Hanumantha Rao S<sup>3</sup>

<sup>1</sup>KL University, Vaddeswaram, Guntur (Dt.), Andhra Pradesh, India

<sup>2</sup>Vignan University, Guntur (DT.), Andhra Pradesh, India

\*Corresponding author: Srinivasa Rao T, KL University, Vaddeswaram, Guntur (Dt.), Andhra Pradesh, India, Tel: 0863239 9999; E-mail: tsr\_2505@kluniversity.in

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### Abstract

The theory of soft sets initiated by Molodtsov, based on soft set theory in this paper we defined the  $\Gamma$ - Soft set,  $\Gamma$ - Soft subset, equality of two  $\Gamma$ - Soft sets, complement of  $\Gamma$ - Soft set, null  $\Gamma$ - Soft set, NOT  $\Gamma$ - Soft set and absolute  $\Gamma$ - Soft set with examples. Also,  $\Gamma$ - Soft binary operations like AND, OR operations along with Union and Intersection of  $\Gamma$ - Soft sets. De-Morgan's laws on  $\Gamma$ - Soft sets and some fundamental results are carried out. In this paper, we introduced the second parameter set,  $\Gamma$  which denotes the name of a brand or company.

*Keywords:  $\Gamma$ - Soft set;  $\Gamma$ - Soft sub; Set not  $\Gamma$ - Soft set; Absolute  $\Gamma$ - Soft set; Complement of  $\Gamma$ - Soft set*

### Introduction

The soft theory was first initiated by Molodtsov [1] as one of the important Mathematical tool to solve the problems with uncertainties. Molodtsov discussed the potential of soft set theory for applications in several directions. Maji et al. [2] studied the detailed theoretical study of soft sets like soft sub sets, NOT set, equality of soft sets. They studied operations soft sets such as Union, Intersection, AND and OR operations. Ahmad and Athar Kharal [3] introduced the arbitrary fuzzy soft union and intersection and proved De-Morgan laws in fuzzy soft theory. Babitha KV and Sunil JJ [4] introduced the notation of soft filters in residuated lattices and their basic properties. They studied relations between soft residuated lattices and soft filter residuated lattices. Cagman et al. [5] studied Fp-soft sets and their operations, t-norm and t-co norm products of Fp- soft sets and their properties. Ibrahim AM and Yusuf AO [6] gave a critical survey of the development of soft set theory and enumerates some of its various applications in different to date. Saakshi Saraf [7] presented the valuable survey on soft set theory and defined some notations on soft sets. Onyeozili IA, Gwary TM [8] were studied the systematic and critical study of the basics of soft set theory, which includes the properties of soft set relations. In this paper, we introduced a new parameter,  $\Gamma$  in Soft set structure, which indicate the name of the company or name of the brand.

## Preliminaries

In this section, we discuss some fundamental definitions and results on  $\Gamma$ -Soft sets with related examples.

$\Gamma$ -Soft set: Let  $U$  be the Universal set and  $P(U)$  be the power set of  $U$ . Let  $K$  and  $\Gamma$  be the sets of parameters attributes. The triode  $(F, L, \Gamma)$  is called a  $\Gamma$ - Soft set over the Universal set,  $U$  is  $(F, L, \Gamma) = \{F(a, \gamma) : a \in L, \gamma \in \Gamma\}$  where  $F$  is a mapping given by  $F: L \times \Gamma \rightarrow P(U)$  and  $L$  is the sub set of  $K$ .

**Example 1:** Let  $U = \{u_1, u_2, u_3, u_4, u_5\}$  be a universal set consisting of a set of six Televisions under consideration. Let  $K = \{e_1, e_2, e_3\}$  and  $\Gamma = \{\gamma_1, \gamma_2\}$  be the sets of parameters with respect to  $U$ , where  $e_1$ : stands for costly,  $e_2$ : stands for cheaper and  $e_3$ : stands for beautiful and also  $\gamma_1$ : stands for brand-1 company,  $\gamma_2$ : stands for brand-2 company.

Let  $L = \{e_1, e_2\} \subseteq K$ . Then  $L \times \Gamma = \{(e_1, \gamma_1), (e_1, \gamma_2), (e_2, \gamma_1), (e_2, \gamma_2)\}$ .

Suppose that  $F(e_1, \gamma_1) = \{u_1, u_2\}$ ,  $F(e_1, \gamma_2) = \{u_2, u_3, u_4\}$ ,  $F(e_2, \gamma_1) = \{u_2, u_5\}$ ,  $F(e_2, \gamma_2) = \{u_3, u_5\}$ .

The  $\Gamma$ - soft set,  $(F, L, \Gamma)$  is a parameterized family  $\{F(e_i, \gamma_j), i = 1,2 \text{ and } j = 1,2\}$  of sub sets of the set  $U$  and gives the collection of approximations given as follows.

$(F, L, \Gamma) = \{(F(e_1, \gamma_1), \{u_1, u_2\}), (F(e_1, \gamma_2), \{u_2, u_3, u_4\}), (F(e_2, \gamma_1), \{u_2, u_5\}), (F(e_2, \gamma_2), \{u_3, u_5\})\}$ .

Where,

$F(e_1, \gamma_1)$  = Brand -1 company costly Televisions =  $\{u_1, u_2\}$

$F(e_1, \gamma_2)$  = Brand -2 company costly Televisions =  $\{u_2, u_3, u_4\}$

$F(e_2, \gamma_1)$  = Brand -1 company cheaper Televisions =  $\{u_2, u_5\}$

$F(e_2, \gamma_2)$  = Brand -2 company cheaper Televisions =  $\{u_3, u_5\}$

$\Gamma$ - Soft sub set: Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a Universal set,  $U$ , we say that  $(F, L, \Gamma)$  is a  $\Gamma$ - Soft sub set of  $(G, M, \Gamma)$  if,

(i)  $L \subseteq M$

(ii) For every  $e \in L, \gamma \in \Gamma, F(e, \gamma), G(e, \gamma)$  are identical approximations, where  $L$  and  $M$  are sub sets of a parameter set  $K$  and  $\Gamma$  is also a parameter set.

**Example 2:** Let  $U = \{u_1, u_2, u_3, u_4, u_5\}$  be a universal set,  $K = \{e_1, e_2, e_3, e_4\}$  and  $\Gamma = \{\gamma_1, \gamma_2\}$  be the sets of parameters with respect to  $U$ .

Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a Universal set,  $U$  where  $L$  and  $M$  are sub sets of  $K$ .

Let  $L = \{e_1, e_2\}$ ,  $M = \{e_1, e_2, e_4\}$  and also the approximations are given as follows.

$F(e_1, \gamma_1) = \{u_1, u_2\}$ ,  $F(e_1, \gamma_2) = \{u_2\}$ ,  $F(e_2, \gamma_1) = \{u_3, u_4\}$ ,  $F(e_2, \gamma_2) = \{u_5\}$  and

$G(e_1, \gamma_1) = \{u_1, u_2, u_3\}$ ,  $G(e_1, \gamma_2) = \{u_2, u_5\}$ ,  $G(e_2, \gamma_1) = \{u_3, u_4\}$ ,  $G(e_2, \gamma_2) = \{u_5, u_1\}$

$G(e_4, \gamma_1) = \{u_1\}$ ,  $G(e_4, \gamma_2) = \{u_2\}$ .

From the above example, we observed that  $L$  is the sub set of  $M$  and  $F$  and  $G$  has identical approximations (TABLE 1).

TABLE 1. Tabular representation of example 2.

U	Costly, Brand-1	Costly, Brand-2	Cheaper, Brand-1	Cheaper, Brand-2
$u_1$	1	0	0	0
$u_2$	1	1	1	0
$u_3$	0	1	0	1
$u_4$	0	1	0	0
$u_5$	0	0	1	1

**Equality of two  $\Gamma$ - Soft sets:** Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a common Universal set,  $U$  are said to be equal if  $(F, L, \Gamma)$  is sub set of  $(G, M, \Gamma)$  and  $(G, M, \Gamma)$  is sub set of  $(F, L, \Gamma)$ .

**3 Operations on  $\Gamma$ - soft sets:** Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a common Universal set,  $U$  where  $L$  and  $M$  are the sub sets of parameters set  $K$ , and  $\Gamma$  is a parameter set.

**‘AND’ Operation on two  $\Gamma$ -Soft sets:** The ‘AND’ operation between two  $\Gamma$ - soft sets  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  is denoted by  $(F, L, \Gamma) \wedge (G, M, \Gamma)$ , it is defined by  $(F, L, \Gamma) \wedge (G, M, \Gamma) = (N, (L \times \Gamma) \times (M \times \Gamma))$ , where  $N((e_1, \gamma_1), (e_2, \gamma_2)) = F(e_1, \gamma_1) \cap G(e_2, \gamma_2)$ ,  $\forall ((e_1, \gamma_1), (e_2, \gamma_2)) \in (L \times \Gamma) \times (M \times \Gamma)$ , in which  $e_1 \in L$ ,  $e_2 \in M$  and  $\gamma_1, \gamma_2 \in \Gamma$ .

**Example 3:** Let  $U = \{u_1, u_2, u_3, u_4\}$  be a universal set consisting of a set of four televisions under consideration,  $L = \{\text{costly television}\}$ ,  $M = \{\text{cheaper television}\}$  and  $\Gamma = \{\text{Brand-1 company, Brand-2 company}\}$  be the sets of parameters with respect to  $U$ .

Let the approximations be:

$F(\text{costly, brand-1 company}) = \{u_1, u_2\}$

$F(\text{costly, brand-2 company}) = \{u_3, u_4\}$

$G(\text{cheaper, brand-1 company}) = \{u_1, u_2, u_3\}$

$$G(\text{cheaper, brand-2 company}) = \{u_2, u_3, u_4\}$$

Then, by AND operation

$$N(\text{costly television, brand-1, cheaper television, brand-2}) = \{u_2\}.$$

TABLE 2. Tabular representation of example 3.

U	Costly, Brand-1	Costly, Brand-2	Cheaper, Brand-1	Cheaper, Brand-2	(Costly, Brand-1, Cheaper, Brand-2)
u <sub>1</sub>	1	0	0	1	0
u <sub>2</sub>	1	0	1	1	1
u <sub>3</sub>	0	1	1	1	0
u <sub>4</sub>	0	1	1	0	0

‘OR’ Operation on two  $\Gamma$ -Soft sets: The ‘OR’ operation between two  $\Gamma$ - soft sets  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  is denoted by  $(F, L, \Gamma) \vee (G, M, \Gamma)$ , it is defined by  $(F, L, \Gamma) \vee (G, M, \Gamma) = (N, (L \times \Gamma) \times (M \times \Gamma))$ , where  $N((e_1, \gamma_1), (e_2, \gamma_2)) = F(e_1, \gamma_1) \cup G(e_2, \gamma_2), \forall ((e_1, \gamma_1), (e_2, \gamma_2)) \in (L \times \Gamma) \times (M \times \Gamma)$ , in which  $e_1 \in L, e_2 \in M$  and  $\gamma_1, \gamma_2 \in \Gamma$  (TABLE 2).

**Example 4:** Let  $U = \{u_1, u_2, u_3, u_4\}$  be a universal set consisting of a set of four televisions under consideration,  $L = \{\text{costly television}\}$ ,  $M = \{\text{cheaper television}\}$  and  $\Gamma = \{\text{Brand-1 company, Brand-2 company}\}$  be the sets of parameters with respect to  $U$ .

Let the approximations be:

$$F(\text{costly, brand-1 company}) = \{u_1, u_2\}$$

$$F(\text{costly, brand-2 company}) = \{u_3, u_4\}$$

$$G(\text{cheaper, brand-1 company}) = \{u_1, u_2, u_3\}$$

$$G(\text{cheaper, brand-2 company}) = \{u_2, u_3, u_4\}$$

Then, by OR operation

$$N(\text{costly television, brand-1, cheaper television, brand-2}) = \{u_1, u_2, u_3, u_4\}.$$

TABLE 3. Tabular representation of example 4.

U	Costly, Brand-1	Costly, Brand-2	Cheaper, Brand-1	Cheaper, Brand-2	(Costly, Brand-1, Cheaper, Brand-2)
u <sub>1</sub>	1	0	0	1	1
u <sub>2</sub>	1	0	1	1	1
u <sub>3</sub>	0	1	1	1	1
u <sub>4</sub>	0	1	1	0	1

**Result:** Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a common Universal set,  $U$  if  $(F, L, \Gamma)$  is the sub set of  $(G, M, \Gamma)$  then it does not imply that every element of  $(F, L, \Gamma)$  is an element of  $(G, M, \Gamma)$  (TABLE 3).

**Proof:** We can verify the above Result by taking an example.

Let  $U = \{u_1, u_2, u_3, u_4, u_5\}$  be a universal set,  $K = \{e_1, e_2, e_3, e_4\}$  and  $\Gamma = \{\gamma_1, \gamma_2\}$  be the sets of parameters with respect to  $U$  and also  $L = \{e_1\}$ ,  $M = \{e_1, e_2\}$  are the sub sets of  $K$ .

Let suppose the following approximations:

$F(e_1, \gamma_1) = \{u_1, u_2\}$ ,  $F(e_1, \gamma_2) = \{u_3, u_5\}$ ,  $G(e_1, \gamma_1) = \{u_1, u_2, u_3\}$ ,  $G(e_1, \gamma_2) = \{u_3, u_5\}$ ,  $G(e_2, \gamma_1) = \{u_1, u_5\}$ ,  $G(e_2, \gamma_2) = \{u_5\}$ .

Therefore,

$F(e_1, \gamma_1) \subset G(e_1, \gamma_1)$

$F(e_1, \gamma_2) \subset G(e_1, \gamma_2)$

i.e., for each  $(e_i, \gamma_j) \in L \times \Gamma$  imply that  $F(e_i, \gamma_j) \subset G(e_i, \gamma_j)$

but,

$(F, L, \Gamma) = \{((e_1, \gamma_1), \{u_1, u_2\}), ((e_1, \gamma_2), \{u_3, u_5\})\}$  and

$(G, M, \Gamma) = \{((e_1, \gamma_1), \{u_1, u_2, u_3\}), ((e_1, \gamma_2), \{u_3, u_5\}), (G(e_2, \gamma_1), \{u_1, u_5\}), (G(e_2, \gamma_2), \{u_5\})\}$

From above it is clear that

Since  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are not containing identical approximations then  $(F, L, \Gamma) \not\subset (G, M, \Gamma)$ .

$\Gamma$ - Not set of a set of parameters: Let  $K = \{e_i/1 \leq i \leq n\}$  and  $\Gamma = \{\gamma_j/1 \leq j \leq n\}$  be two sets of parameters. The  $\Gamma$ -NOT set of  $E \times \Gamma$  is denoted by  $(K \times \Gamma)$  and is defined as  $(K \times \Gamma) = \{(e_i, \gamma_j), 1 \leq i, j \leq n\}$ , where

$(e_i, \gamma_j) = (\bar{e}_i, \bar{\gamma}_j)$ , in which  $\bar{e}_i$  means not  $e_i$  and  $\bar{\gamma}_j$  means not  $\gamma_j$ .

**Example 5:** Let the parameters set are  $K = \{\text{costly, cheaper}\}$ ;  $\Gamma = \{\text{brand-1, brand-2}\}$

Then,  $K = \{\text{Not costly, Not cheaper}\}$ ;  $\Gamma = \{\text{Not brand-1, Not brand-2}\}$ .

The following Propositions are obvious

Proposition:

$$((K \times \Gamma)) = K \times \Gamma$$

$$((L \times \Gamma) \cup (M \times \Gamma)) = (L \times \Gamma) \cup (M \times \Gamma)$$

$$((L \times \Gamma) \cap (M \times \Gamma)) = (L \times \Gamma) \cap (M \times \Gamma)$$

Where  $K$  and  $\Gamma$  are sets of parameters over a common universal set  $U$  and  $L, M, K$ .

**Complement of a  $\Gamma$ - Soft set:** The complement of a  $\Gamma$ - soft set  $(F, L, \Gamma)$  over a universal set,  $U$  is denoted by  $(F, L, \Gamma)^c$  and is defined as  $(F, L, \Gamma)^c = (F^c, L, \Gamma)$ , where

$F^c: ((L \times \Gamma) \rightarrow P(U))$ , is a mapping given by  $F^c(e, \gamma) = U - F(e, \gamma), \forall e \in L, \gamma \in \Gamma$ .

**Example 6:** Let  $U = \{u_1, u_2, u_3, u_4\}$  be a universal set consisting of a set of four A. C.'s under consideration and  $L = \{e_1 = \text{beautiful}\}$  and  $\Gamma = \{\gamma_1 = \text{brand -1 company}, \gamma_2 = \text{brand -2 company}\}$  are the parameter sets corresponding to  $U$ .

$(F, L, \Gamma) = \{F(e_1, \gamma_1) = \text{brand -1 beautiful A. C.'s} = \{u_1, u_2\}, F(e_1, \gamma_2) = \text{brand -2 beautiful A. C.'s} = \{u_4\}\}$ .

The complement of a  $\Gamma$ - soft set  $(F, L, \Gamma)$  is given by

$(F, L, \Gamma)^c = \{F(e_1, \gamma_1) = \text{Not brand -1 company and Not beautiful A. C.'s} = \{u_3, u_4\}, F(e_1, \gamma_2) = \text{Not brand -2 company and Not beautiful A. C.'s} = \{u_1, u_2, u_3\}\}$ .

**Null  $\Gamma$ - soft set:** A  $\Gamma$ - soft set  $(F, L, \Gamma)$  over a universal set,  $U$  is said to be a Null  $\Gamma$ - soft set if  $F(e, \gamma) = \emptyset, \forall e \in L, \gamma \in \Gamma$ , where is null set.

**Example 7:** Let  $U = \{u_1, u_2, u_3, u_4\}$  be a universal set consisting of a set of four televisions under consideration,  $L = \{e_1, e_2\}$  and  $\Gamma = \{\gamma_1, \gamma_2\}$  be the sets of parameters with respect to  $U$ , where  $e_1$ : stands for 'picture tube',  $e_2$ : stands for 'speakers',  $\gamma_1$ : stands for 'brand -1 company',  $\gamma_2$ : stands for 'brand -2 company'.

The  $(F, L, \Gamma)$  describes the assembled televisions and the  $\Gamma$ - soft set  $(F, L, \Gamma)$  is defined as follows

$F(e_1, \gamma_1) = \text{Brand-1 company Picture tube assembled Televisions.}$

$F(e_1, \gamma_2) = \text{Brand-2 company Picture tube assembled Televisions.}$

$F(e_2, \gamma_1) = \text{Brand-1 company Speakers assembled Televisions.}$

$F(e_2, \gamma_2) = \text{Brand-2 company Speakers assembled Televisions.}$

$(F, L, \Gamma) = \{F(e_1, \gamma_1) = \emptyset, F(e_1, \gamma_2) = \emptyset, F(e_2, \gamma_1) = \emptyset, F(e_2, \gamma_2) = \emptyset\}$ .

Hence  $(F, L, \Gamma) = \emptyset$ , is Null  $\Gamma$ - soft set.

**Proposition:** Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a common Universal set,  $U$  Then,

$$((F, L, \Gamma) \wedge (G, M, \Gamma))^c = (F, L, \Gamma)^c \wedge (G, M, \Gamma)^c$$

$$((F, L, \Gamma) \vee (G, M, \Gamma)) c = (F, L, \Gamma) c \vee (G, M, \Gamma) c$$

**Proof:** we have to prove  $((F, L, \Gamma) \wedge (G, M, \Gamma)) c = (F, L, \Gamma) c \wedge (G, M, \Gamma) c$

By definition,  $(F, L, \Gamma) \wedge (G, M, \Gamma) = (N, (L \times \Gamma) \times (M \times \Gamma))$

$$((F, L, \Gamma) \wedge (G, M, \Gamma)) c = (N, (L \times \Gamma) \times (M \times \Gamma)) c = (Nc, (L \times \Gamma) \times (M \times \Gamma))$$

Where,  $Nc: ((L \times \Gamma) \times (M \times \Gamma)) \rightarrow P(U)$ , mapping is defined by  $Nc((a, \gamma), (b, \gamma)) = U - N((a, \gamma), (b, \gamma))$ ,  $a \in L, b \in M$  and  $\gamma \in \Gamma$ .

Now,  $(F, L, \Gamma) c \wedge (G, M, \Gamma) c = (Nc, (L \times \Gamma)) \cap (Nc, (M \times \Gamma))$

$$= (J, (L \times \Gamma) \times (M \times \Gamma))$$

Where  $J((a, \gamma), (b, \gamma)) = Fc(a, \gamma) \wedge Gc(b, \gamma)$

Now consider  $(a, \gamma), (b, \gamma) \in (L \times \Gamma) \times (M \times \Gamma)$

$Fc(a, \gamma) \wedge Gc(b, \gamma) = U - N((a, \gamma), (b, \gamma))$

$$= U - (F(a, \gamma) \cup G(b, \gamma))$$

$$= U - F(a, \gamma) \cap G(b, \gamma)$$

$$= Fc((a, \gamma)) \cap Gc((b, \gamma))$$

$$= J((a, \gamma), (b, \gamma)).$$

Therefore  $((F, L, \Gamma) \wedge (G, M, \Gamma)) c = (F, L, \Gamma) c \wedge (G, M, \Gamma) c$ .

we have to prove,  $((F, L, \Gamma) \vee (G, M, \Gamma)) c = (F, L, \Gamma) c \vee (G, M, \Gamma) c$

By definition,  $(F, L, \Gamma) \vee (G, M, \Gamma) = (N, (L \times \Gamma) \times (M \times \Gamma))$

Now,  $(F, L, \Gamma) c \vee (G, M, \Gamma) c = (Nc, (L \times \Gamma)) \cup (Nc, (M \times \Gamma))$

$$= (H, ((L \times \Gamma) \times (M \times \Gamma)))$$

Where  $H((a, \gamma), (b, \gamma)) = Fc(a, \gamma) \cup Gc(b, \gamma)$

Let  $((a, \gamma), (b, \gamma)) \in ((L \times \Gamma) \times (M \times \Gamma))$

Consider  $Hc((a, \gamma), (b, \gamma)) = U - H((a, \gamma), (b, \gamma))$

$$= U - (F(a, \gamma) \cap G(b, \gamma))$$

$$\begin{aligned}
 &= (U - F(a, \gamma)) \cup (U - G(b, \gamma)) \\
 &= Fc((a, \gamma)) \cup Gc((b, \gamma)) \\
 &= H(((a, \gamma), (b, \gamma)))
 \end{aligned}$$

Nc and H represents the same.

Hence the theorem is proved.

Absolute  $\Gamma$ - soft set: A  $\Gamma$ - soft set  $(F, L, \Gamma)$  over a universal set, U an Absolute  $\Gamma$ - soft set is denoted by and it is defined as if  $\forall (e, \gamma) \in L, F(e, \gamma) = U$ .

Clearly  $(F, L, \Gamma) = (U, L, \Gamma)$ .

**Example 8:** Let  $U = \{u_1, u_2, u_3, u_4\}$  be a universal set consisting of a set of four televisions under consideration,  $L = \{e_1, e_2\}$  and  $\Gamma = \{\gamma_1, \gamma_2\}$  be the sets of parameters with respect to U, where  $e_1$ : stands for ‘picture tube’,  $e_2$ : stands for ‘speakers’,  $\gamma_1$ : stands for ‘brand -1 company’,  $\gamma_2$ : stands for ‘brand -2 company’.

The  $\Gamma$  – soft set  $(F, L, \Gamma)$  describes the “The Assembled Televisions”.

The  $\Gamma$ – soft set is defined as follows:

$L(e_1, \gamma_1) =$  not brand – 1, not picture tube Assembled TV’s.

$L(e_1, \gamma_2) =$  not brand – 2, not picture tube Assembled TV’s.

$L(e_2, \gamma_1) =$  not brand – 1, not speaker Assembled TV’s.

$L(e_2, \gamma_2) =$  not brand – 2, not speaker Assembled TV’s.

The  $\Gamma$ – soft set  $(F, L, \Gamma)$  is the collection of approximations as below:

$(F, L, \Gamma) = \{(L(e_1, \gamma_1), \{u_1, u_2, u_3, u_4\}), (L(e_1, \gamma_2), \{u_1, u_2, u_3, u_4\}), (L(e_2, \gamma_1), \{u_1, u_2, u_3, u_4\}), (L(e_2, \gamma_2), \{u_1, u_2, u_3, u_4\})\}$ .

Hence  $(F, L, \Gamma) = U$ .

$\Gamma$ -soft set Operations:

Union of two  $\Gamma$ -soft sets: Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a common Universal set, U then the union of these two  $\Gamma$ - soft sets is denoted by  $(F, L, \Gamma) \cup (G, M, \Gamma)$  and is defined by

$$(F, L, \Gamma) \cup (G, M, \Gamma) = (N, (L \times \Gamma) \cup (M \times \Gamma))$$

Where,  $N(e, \gamma) = \{F(e, \gamma) \text{ if } (e, \gamma) \in (L \times \Gamma) - (M \times \Gamma),$



$$= \{G(e, \gamma) \text{ if } (e, \gamma) \in (M \times \Gamma) - (L \times \Gamma),$$

$$= \{F(e, \gamma) \cup G(e, \gamma) \text{ if } (e, \gamma) \in (L \times \Gamma) \cap (M \times \Gamma)$$

**Example 9:** Let  $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$  be a universal set,  $L = \{e_1, e_2\}$ ,  $G = \{e_1, e_2, e_3\}$  and  $\Gamma = \{\gamma_1, \gamma_2\}$  be the sets of parameters with respect to  $U$ . The approximations are given by  $(F, L, \Gamma) = \{F(e_1, \gamma_1), F(e_1, \gamma_2), F(e_2, \gamma_1), F(e_2, \gamma_2)\}$  and  $(G, M, \Gamma) = \{G(e_1, \gamma_1), G(e_1, \gamma_2), G(e_2, \gamma_1), G(e_2, \gamma_2), G(e_3, \gamma_1), G(e_3, \gamma_2)\}$ .

Suppose that:

$$F(e_1, \gamma_1) = \{u_1, u_2\}, F(e_1, \gamma_2) = \{u_1, u_2, u_3\}, F(e_2, \gamma_1) = \{u_3, u_4, u_5\}, F(e_2, \gamma_2) = \{u_6\}$$

$$G(e_1, \gamma_1) = \{u_3\}, G(e_1, \gamma_2) = \{u_4, u_5\}, G(e_2, \gamma_1) = \{u_6\}, G(e_2, \gamma_2) = \{u_5, u_6\}, G(e_3, \gamma_1) = \{u_2, u_4\}, G(e_3, \gamma_2) = \{u_4, u_5\}.$$

$$(F, L, \Gamma) \cup (G, M, \Gamma) = \{(G(e_3, \gamma_1), \{u_2, u_4\}), (G(e_3, \gamma_2), \{u_4, u_5\})\}.$$

Intersection of two  $\Gamma$  – soft sets: Let  $(F, L, \Gamma)$  and  $(G, M, \Gamma)$  are two  $\Gamma$ - soft sets over a common Universal set,  $U$  then the union of these two  $\Gamma$ - soft sets is denoted by  $(F, L, \Gamma) \cap (G, M, \Gamma)$  and is defined by

$$(F, L, \Gamma) \cap (G, M, \Gamma) = (N, (L \times \Gamma) \cap (M \times \Gamma)), \text{ where } N(e, \gamma) = F(e, \gamma) \text{ or } G(e, \gamma) \text{ for every } (e, \gamma) \in (L \times \Gamma) \cap (M \times \Gamma).$$

**Example 10:** Let  $U = \{u_1, u_2, u_3, u_4\}$  be a universal set,  $L = \{e_1, e_2, e_3\}$ ,  $G = \{e_1, e_2\}$  and  $\Gamma = \{\gamma_1, \gamma_2\}$  be the sets of parameters with respect to  $U$ . The approximations are given by

$$(G, L, \Gamma) = \{G(e_1, \gamma_1), G(e_1, \gamma_2)\} \text{ and } (F, M, \Gamma) = \{F(e_1, \gamma_1), F(e_1, \gamma_2), F(e_2, \gamma_1), F(e_2, \gamma_2), F(e_3, \gamma_1), F(e_3, \gamma_2), G(e_1, \gamma_1), G(e_1, \gamma_2)\}.$$

$$\text{Where, } F(e_1, \gamma_1) = \{u_1\}, F(e_1, \gamma_2) = \{u_1, u_2\}, F(e_2, \gamma_2) = \{u_2\}, F(e_3, \gamma_1) = \{u_3, u_4\}, F(e_3, \gamma_2) = \{u_4\}, G(e_1, \gamma_1) = \{u_1\}, G(e_1, \gamma_2) = \{u_1, u_2\}.$$

$$(F, L, \Gamma) \cap (G, M, \Gamma) = \{F(e_1, \gamma_1), F(e_1, \gamma_2)\}, \text{ or } \{G(e_1, \gamma_1), G(e_1, \gamma_2)\}.$$

$$\text{i.e., } N(e_1, \gamma_1) = \{u_1\}, N(e_1, \gamma_2) = \{u_1, u_2\}.$$

For Some operations on properties of  $\Gamma$ - soft sets: In this paper, we will state without proofs the basic properties on  $\Gamma$ - soft set operations and these operations have been established by many authors.

**(i) Idempotent properties:**

$$(a) (F, L, \Gamma) \cup (F, L, \Gamma) = (F, L, \Gamma)$$

$$(b) (F, L, \Gamma) \cap (F, L, \Gamma) = (F, L, \Gamma)$$

**(ii) Identity properties:**

$$(a) (F, L, \Gamma) \cup \phi = (F, L, \Gamma)$$

$$(b) (F, L, \Gamma) \cap U = (F, L, \Gamma)$$

$$(c) (F, L, \Gamma) - \varphi = (F, L, \Gamma)$$

$$(d) (F, L, \Gamma) - (F, L, \Gamma) = \varphi.$$

**(iii) Domination properties:**

$$(a) (F, L, \Gamma) \cup U = U$$

$$(b) (F, L, \Gamma) \cap \varphi = (F, L, \Gamma)$$

**(iv) Complementation properties:**

$$(a) \varphi^c = U$$

$$(b) U^c = \varphi$$

**(v) Commutative property:**

$$(a) (F, L, \Gamma) \cup (G, M, \Gamma) = (G, M, \Gamma) \cup (F, L, \Gamma)$$

$$(b) (F, L, \Gamma) \cap (G, M, \Gamma) = (G, M, \Gamma) \cap (F, L, \Gamma)$$

**Conclusion**

In this paper, we studied the fundamentals  $\Gamma$ - soft sets such as  $\Gamma$ - Soft subset, equality of two  $\Gamma$ - Soft sets, complement of  $\Gamma$ - Soft set, null  $\Gamma$ - Soft set, NOT  $\Gamma$ - Soft set and absolute  $\Gamma$ - Soft set with examples. Also,  $\Gamma$ - Soft binary operations like AND, OR operations along with Union and Intersection of  $\Gamma$ - Soft sets. De-Morgan's laws on  $\Gamma$ - Soft sets. We can extend this work to related concepts like fuzzy  $\Gamma$ - soft sets,  $\Gamma$ - soft semi groups.

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